P425/1
PURE
MATHEMATICS
Paper 1
July/ August 2022

3hours

KAMSSA JOINT MOCK EXAMINATIONS

Uganda Advanced Certificate Of Education

PURE MATHEMATICS

Paper 1
3hours

Instructions to candidates:

- *Answer **All** the **eight** questions in section **A** and **five** questions from section **B**.*
- *Any additional question **(s)** answered will not be marked.*
- ***All** working **must be** shown clearly.*
- *Begin each answer on a fresh page.*
- *Graph paper is provided.*
- ***Silent non-programmeable, scientific calculators and mathematical tables with** atleast of formaulae may be used.*
- *State the degree of accuracy at the end of each answer given. If a calculator or a mathematical table is used , indicate **Cal** for calculator or **Tab** for mathematical tables.*

SECTION A (40 MARKS)

Answer all questions in this section

1. Solve for y: $4cosy = 3tany + 3secy$ for $0^0 \leq y \leq 360^0$ **(5 marks)**

2. Evaluate: $\int_0^{\frac{\pi}{2}} xsin2x\ dx$ **(5 marks)**

3. Solve for ***t*** in : $5^{2t} = 5^{t+1} - 6$ **(5 marks)**

4. Show that the locus of a point ***P(x,y)*** which moves such that it divides the line joining $A(2,-3)$ and $B(3,4)$ in the ration 1:2 is a circle. State its radius and centre.(5 marks)

5. Given that $y = \sqrt{(4 + 3sinx)}$, show that :

$$2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + y^2 = 4$$ **(5 marks)**

6. Find the perpendicular distance of the point $P(3,-1,2)$ from the line

$r = i + j + 3k + \mu(2i + 4j - k)$. **(4 marks)**

7. Find the values of ***k*** for which the quadratic equations $x^2 + kx - 6k = 0$ and $x^2 - 2x - k = 0$ have a common root. **(6 marks)**

8. Air is pumped into a spherical balloon at a rate of $200cm^3s^{-1}$. When the radius of the balloon is 80mm, find the rate at which the surface area is increasing**.** **(5 marks)**

SECTION B: (60 MARKS)

*Answer any **five** questions from this section. All questions carry equal marks*

9. Evaluate the following:

a. $\int \frac{1}{e^{2x}-1}\,dx$

b. $\int_0^{\frac{\pi}{2}} \frac{1}{1+\cos t}\,dt$ **(12 marks)**

10. a. Find the coefficient of x^3 in the expansion of $\left(\frac{1}{x^2}-x\right)^{18}$ **(5 marks)**

b. Show that $\sqrt{\left(\frac{1+x}{1-x}\right)} = 1 + x + \frac{1}{2}x^2 + \cdots$ and hence using $x = \frac{1}{7}$, show that $\sqrt{3} \approx \frac{196}{113}$. **(7 marks)**

11. a. Given that $2A + B = 45^0$, Show that

$$tanB = \frac{1-2tanA-\tan^2 A}{1+2tanA-\tan^2 A}$$ **(6 marks)**

b. Find the value of x in: $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$ **(6 marks)**

12. a. Use small changes to evaluate $tan\,61^0$ to 2dps. **(5 marks)**

b. Show that $\frac{d}{dx}(cosecx) = -cosecxcotx$ from first principles. **(7 marks)**

13. a. Given the equation of curve as $x^2 + 4x - 8y - 4 = 0$.

i. Show that the curve is a parabola.

ii. Find the coordinates of the vertex. **(4 marks)**

b. Show that if the line $y = mx + c$ touches the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then $c^2 = a^2m^2 + b^2$.

Find the equations of the tangents to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, which are parallel to $y = x + 1$.

(8 marks)

14. Describe the locus of a complex number $z = x + yi$ which moves in the complex plane such that $\arg\left(\frac{z-3}{z-2i}\right) = \frac{\pi}{4}$. **(12 marks)**

15. a. Find the cartesian equation of the plane $r = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 6 \\ -2 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 3 \\ -7 \end{pmatrix}$, given that $R(x, y, z)$ is a general coordinate in the plane. **(7 marks)**

b. Find the cartesian equation of the line which passes through the point $A(4,4,-1)$ and is perpendicular to the plane in (a) above. **(5 marks)**

16. The rate at which the temperature of a liquid in an un-covered pan falls is directly proportional to the difference between the temperature of the liquid and that of the surrounding. The temperature of the liquid is initially 50^0C. After 20 minutes, the temperature of the liquid is 35^0C. Given that the temperature of the surrounding is 15^0C. What will be the temperature of the liquid after 26 minutes? (give your answer to 1 decimal place) **(12 marks)**

END